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In Initial Model for the Finite Displacement response Characteristics of a Fluidyne Pump

Tadéusz M. Drzewiecki



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U.S. Army Electronics Research and Development Command Harry Diamond Laboratories Adelphi, MD 20783

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The literature is particularly sparse on the subject; hence, only the basic references are cited. The report presents a detailed mathematical model of the fluid mechanical and thermodynamic processes occurring during oscillation. A positive-feedback simulation model is postulated that demonstrates, for the first time, physically why there is an onset of oscillations and subsequent sustained motion. This model is based on actual observations of the startup process in a prototype pump.

The initially reported thermodynamic efficiency of this heat engine was less than 0.3 percent. Currently, Fluidyne pumps have demonstrated efficiencies of about 2 percent. The peak efficiencies, however, may theoretically approach 10 percent. The mathematical model has shown that large losses occur in the heat transfer to the working gas, heat losses to the displacer liquid, and friction in the output line. Improvements in these areas may dramatically improve the observed overall efficiency. The efficiency of the cycle itself, that is, the work done by the thermodynamic cycle relative to the output work, is on the order of 10 percent. Finally, the cycle is shown to be close to a Carnot cycle as opposed to the originally supposed Stirling cycle. This last observation has considerable and significant ramifications as far as future analysis is concerned.



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1. INTRODUCTION

The introduction of the Fluidyne "no-moving-parts" pump in 1971^{1,2} stimulated some new interest in the conversion of low grade, low temperature heat energy into useful work. The basic principle was reported to be a closed Stirling cycle, wherein two adjacent U-tubes of liquid were stimulated into oscillation by the application of heat to a connecting U-tube of gas. The open U-tube of oscillating liquid thus was the source of power to do work. A linearized lossless model was proposed by Elrod³ that predicted the onset of oscillation and the frequency thereof. No satisfactory physical explanation was offered for the phenomenon, however.

With the coming of the energy crisis, initial studies on reclaiming waste heat were proposed by the Harry Diamond Laboratories using the Fluidyne. With the proposed increased dependence on nuclear power, additional interest in the Fluidyne became apparent when it was conceptually demonstrated that a Fluidyne could serve as an independent power source for fluidic monitoring and control schemes that are completely decoupled from standard electromechanical systems in use. Such incidents as the Brown's Ferry fire and the loss of secondary controls could be avoided by using the residual nuclear heat to operate a self-contained Fluidyne pump and fluidic sensors and controls operating within the pressure vessel itself. A system having no moving parts except the working fluid would be extremely reliable.

The poor efficiency of currently available³ pumps allows profitable operation only where the heat source costs are negligible or, as in a nuclear plant, not of concern. Improvement of the efficiency would allow greater utilization and would decrease the unit size per unit output power. This decrease would allow the use of smaller devices in such applications as nuclear power plant auxiliary control systems.

This report presents a comprehensive basis for the understanding of the Fluidyne pump's operation, analyzes the concept of the efficiency of the thermodynamic cycle, and provides important guidelines for design.

2. PHYSICAL DESCRIPTION OF FLUIDYNE STARTUP

In the schematic diagram (fig. 1), a reservoir of liquid is shown with a U-tube of gas connecting two liquid columns. A separate U-tube filled with the liquid is situated with one end directly beneath one end of the gas-filled tube. The other end, for this analysis, is left open to the atmosphere. The free surface motion indicates the output mechanical work available.

The gas volume may be thought to be composed of two imaginary halves. The right half, labeled "2," which connects to the output tube, is the hot side; the left half, labeled "1," is the cold side. Heat is continually applied to the right half, and the left side is maintained at a lower or ambient temperature. In the experiments conducted, the cold side was cooled by natural convection to ambient.

C. West, The Fluidyne Heat Engine, United Kingdom Atomic Energy Authority, Harwell, UK, AERE-R-6775 (1971).

²C. West, And Yet It Moves, New Scientist, 29 (August 1974).

³H. Elrod, The Fluidyne Heat Engine--How to Build One, How it Works, National Technical Information Service, Springfield, VA, AD A006367 (1974).

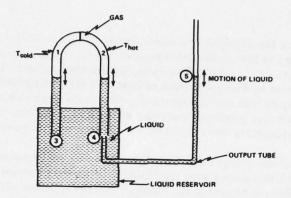


Figure 1. Schematic of Fluidyne pump.

The startup and subsequent sustained oscillations may be described physically in five steps (fig. 2).

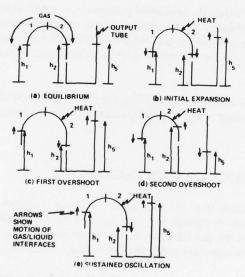


Figure 2. Schematic representation of Fluidyne startup process.

a. In equilibrium conditions, no heat is applied. The liquid levels h_1 , h_2 , and h_5 are prescribed by the static pressure in the gas volume of columns (1) and (2), and $h_1 = h_2$. (If the gas pressure p_{12} is ambient, p_{∞} , then all three levels are equal.)

- b. Heat is applied to the right half of the gas volume of column (2). The gas expands, and the gas volume, V_{ij} increases while the gas pressure p_{12} also increases. As a natural consequence, liquid levels h_1 and h_2 decrease, and liquid level h_3 increases as the liquid in the displacer is accelerated. The gas temperature rises in column (2).
- c. The inertia of the liquid in the output tube keeps the liquid moving; hence, the liquid in column (2) tends to keep moving downward, also. As more hot side is exposed, the liquid is accelerated further until equilibrium in the gas is reached.
- d. Gravity stops the upward motion of liquid level h_1 , and levels h_2 and h_5 tend to seek equality. Hence h_2 increases, and h_5 starts to decrease. At the same time, there is a reduction of hot volume. This reduction starts an overall gas volume decrease due to a temperature reduction since less gas is exposed to the heat source. The pressure p_{12} decreases. All this action is augmented by the descent of the output level due to gravity, which produces a dynamic head at the base of column (2), further increasing h_2 . An overshoot of the two liquid levels occurs, $h_2 > h_1$, $h_2 > h_5$. This overshoot gives rise again to a gravity potential between liquid levels.
- e. The liquid levels again seek equilibrium with level h₂ decreasing. This decrease exposes more of the gas to the heat source and causes the gas to expand; the cycle begins all over again, with the output liquid again moving upward.

This is the physical description as observed in a glass or plastic demonstration model built and tested during this program.

3. MATHEMATICAL MODEL

Geisow⁴ has proposed a set of governing equations for a lossless Fluidyne system in an attempt to compute the conditions required for the onset of instability. To this end, he used the Euler equations of motion in one dimension, linearized them, and then performed a first-order perturbation stability analysis. The results of this analysis led to an expression for the critical temperature differential, ΔT , as a function of the pump geometry and for the oscillating frequency, ω . This is given as equations (1) and (1a):

$$\frac{\Delta T}{T_1 + T_2} = \frac{\rho g V_e}{2 A_D^P_{\infty}} \left(\frac{\ell_c - \ell_h}{\ell_D} \right) , \qquad (1)$$

$$\omega = \left[2g / \left(2 \ell_h + \ell_D A_c / A_D \right) \right]^{\frac{1}{2}} , \qquad (1a)$$

⁴A. D. Geisow, The Onset of Oscillations in a Lossless Fluidyne, United Kingdom Atomic Energy Authority, Harwell, UK, AERE-M-2840 (October 1976).

where

 T_1 = cold temperature (K), T_2 = hot temperature (K), ρ = liquid density (kg/m³), g = acceleration of gravity (m/s²), V_e = equilibrium volume of gas (m³), A_D = cross-sectional area of displacer tube (m²), P_∞ = atmospheric pressure (Pa), ℓ = equilibrium length of cold leg to output (m), ℓ = equilibrium length of hot leg to output (m), ℓ = length of displacer (m), ℓ = cross-sectional area of cold leg.

This analysis was performed for the geometry shown in figure 3 and indicates a strong dependency of system response on the displacer tube.

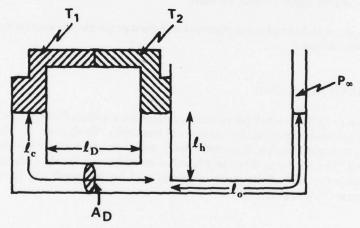


Figure 3. Schematic of Fluidyne II used by Geisow (AERE-M-2840, October 1976).

The original Fluidyne proposed by West¹ and described by Elrod³ did indeed have such a configuration; however, currently commercially available Fluidynes⁵ replace the displacer tube with a fluid reservoir as shown in figures 1 and 4.

¹C. West, The Fluidyne Heat Engine, United Kingdom Atomic Energy Authority, Harwell, UK, AERE-R-6775 (1971).

³H. Elrod, The Fluidyne Heat Engine--How to Build One, How it Works, National Technical Information Service, Springfield, VA, AD A006367 (1974).

^{*}Stirling-Cycle Engines Developed at Harwell--The Fluidyne 3 Pump, United Kingdom Atomic Energy Authority, Harwell, UK, Harwell Design Studios 56/1975 (1975).

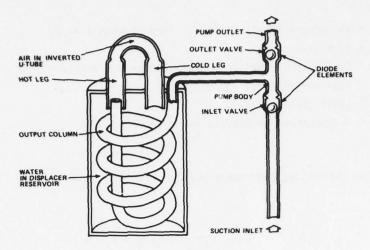


Figure 4. Schematic of actual Fluidyne pump (diode elements—ball valves—permit unidirectional flow).

Interestingly enough, the models shown by Elrod³ always have the output tube axially in line with the hot leg as opposed to being at right angles as shown in figure 3, even when the displacer is present. This leads to the observation of section 2 that dynamic head feedback from the output to the hot leg is important in the pump operation in that the inertia of the moving fluid in the output leg preferentially acts to increase the head in the hot leg.

The following set of governing equations may be generated, assuming slug flow in all tubes for the geometry of figure 1 or 4.

The pressure drop, $\Delta P_{ij} = P_i - P_j$, for any tube is given by the electrical analog equation for time varying slug flow⁶ (derived from the Euler equation):

$$\Delta P_{ij} = L_{ij} \frac{dQ_{ij}}{dt} + R_{ij}Q_{ij} + h_{i}\rho_{L}g$$
 (2)

where

 L_{ij} = liquid inductance (kg/m⁴) between points i and j, = $\rho_L^{\ell}_{ij}/A_{ij}$, ρ_L = liquid density (kg/m³), ℓ_{ij} = tube length between i and j (m), A_{ij} = tube cross-sectional area (m²),

³H. Elrod, The Fluidyne Heat Engine--How to Build One, How it Works, National Technical Information Service, Springfield, VA, AD A006367 (1974).

⁶J. M. Kirshner, Fluid Amplifiers, McGraw-Hill Book Co, New York (1967).

 Q_{ij} = volumetric flow rate (m³/s) in direction i to j, t = time (s), R_{ij} = liquid resistance (for laminar flow R = $8\pi\mu\ell/A^2$) [kg/(m⁴·s)], μ = liquid viscosity [kg/(m·s)], μ = liquid height (m), μ = acceleration of gravity (m/s²).

Therefore, for the cold leg, between steps (a) and (c) the pressure drop is

$$p_3 - p_1 = L_{13} \frac{dQ_{31}}{dt} + R_{31}Q_{31} + h_1\rho_L g$$
 (3)

Similarly, the hot leg pressure drop can be written when the additional total pressure feedback term is included due to the dynamic head effect of the output leg:

$$p_{4} - p_{2} = L_{42} \frac{dQ_{42}}{dt} + R_{42}Q_{42} + h_{2}\rho_{L}g - \frac{\rho_{L}}{2} K_{1} \left(\frac{Q_{45}}{A_{45}}\right)^{2}$$
, (4)

where

$$K_1 = 0$$
, $Q_{45} > 0$; $K_1 = 1$, $Q_{45} < 0$.

Finally, the output leg presure drop is, with hot leg feedback,

$$p_{4} - p_{5} = L_{45} \frac{dQ_{45}}{dt} + R_{45} Q_{45} + h_{5} \rho_{L} g - \frac{\rho_{L}}{2} K_{2} \left(\frac{Q_{42}}{A_{42}}\right)^{2}, \qquad (5)$$

where

$$K_2 = 0$$
, $Q_{42} > 0$; $K_2 = 1$, $Q_{42} < 0$.

The equation relating the liquid height and the volume flow rate is the constitutive equation

$$A \frac{dh}{dt} = Q . (6)$$

Hence, the individual time rates of height change are

$$\frac{dh_1}{dt} = \frac{Q_{31}}{A_{31}} , (7)$$

$$\frac{dh_2}{dt} = \frac{Q_{42}}{A_{42}} \quad , \tag{8}$$

$$\frac{\mathrm{dh}_5}{\mathrm{dt}} = \frac{Q_{45}}{A_{45}} \quad . \tag{9}$$

It follows, from conservation of mass of an incompressible liquid into the reservoir, that the net flow must be zero, or

$$-Q_{31} - Q_{42} - Q_{45} = 0$$
 (10)

Note the minus signs since each flow is assumed to be positive out of the reservoir. The pressure at the base of the cold leg is the reservoir pressure. If the output leg does not impede flow from the reservoir into it or the hot leg, then $p_3 = p_4$. If there is an orifice resistance, then

$$p_{4} - p_{3} = \frac{\rho_{L}Q_{31}^{2}}{2(A_{42} - A_{45})^{2} c_{d}^{2}} , \qquad (11)$$

where cd is the discharge coefficient.

For convenience of this initial analysis, consider p₅ to be ambient:

$$P_5 = 101 \text{ kPa (absolute)}$$
 (12)

We have up to this point 9 equations and 11 unknowns—namely, 5 pressures, 3 flows, and 3 heights; consequently, more equations are required.

If the assumption is made that the pressure is constant at any instant of time in the gas volume, which is a reasonable assumption considering that the process is relatively slow, the gaseous resistance of the tubing is negligible, and the head potential is likewise negligible, then another equation is available:

$$p_1 = p_2 . (13)$$

The gas mass must be conserved so that

$$m_1 + m_2 = constant . (14)$$

This equation has added two unknowns, m_1 and m_2 , the masses of the cold and hot sides of the gas volumes (kg), respectively; hence, there are now 11 equations and 13 unknowns. The equation of state may be written for each side of the gas volume. If it is assumed that the gas is ideal, then

$$p_1 V_1 = m_1 R T_1 \quad , \tag{15}$$

$$p_1 V_2 = m_2 R T_2 \quad , \tag{16}$$

where

$$V_1$$
 = volume of cold gas (m³),
 V_2 = volume of hot gas (m³),
 T_1 = temperature of cold gas (K),
 T_2 = temperature of hot gas (K),
 R = gas constant $\left[m^3/(s^2K)\right]$.

There are now 13 equations and 17 unknowns, the new unknowns being volumes V_1 and V_2 and temperatures T_1 and T_2 .

The gas volumes are directly related geometrically to the position of the gas-liquid interface so that

$$V_1 = \frac{1}{2}V_e - (h_1 - h_e)A_{13} (17)$$

and

$$V_2 = \frac{1}{2}V_e - (h_2 - h_e)A_{24} , \qquad (18)$$

where

h = equilibrium height of liquid (m)

This now brings us to 15 equations and 17 unknowns.

At this point, a statement can be made about temperatures T₁ and T₂. Elrod³ and Geisow⁴ chose to assume that the temperatures are known constants. This choice is tenuous since it requires that a step distribution in temperature exists in the gas tube. For the sake of simplicity, assume that

$$T_1 = constant$$
, (19)

$$T_2 = constant$$
, (20)

where T_1 and T_2 are average temperatures (K) in each volume of gas. These then bring the count of unknowns and equations even at 17 apiece. The evaluation of T_1 and T_2 is given in appendix A. Upon examination of the governing equations, one notes that the pressure drop equations are nonlinear since the inductance and the resistance terms are functions of the length of the liquid, which is variable. Linearization can be achieved by stipulating that the total excursions of the liquid interfaces be considerably smaller than the equilibrium lengths. For computer solution, however, this stipulation is not necessary. There are five independent first-order ordinary differential equations, since one of the flow equations is used to find a pressure. The solution of the entire set of equations gives the displacement of the output column of liquid. Since the object of this study is to establish the parameters relating to the efficiency of the pump, a quantity of interest is the output power, W. This is the time rate of increase of potential energy of the output or the root mean square (rms) value of the product of the flow and the dynamic head. The instantaneous power out is

$$W = \frac{1}{2} \left(\frac{Q_{45}}{A_{45}} \right)^2 Q_{45} . \tag{21}$$

³H. Elrod, The Fluidyne Heat Engine--How to Build One, How it Works, National Technical Information Service, Springfield, VA, AD A006367 (1974).

⁴A. D. Geisow, The Onset of Oscillations in a Lossless Fluidyne, United Kingdom Atomic Energy Authority, Harwell, UK, AERE-M-2840 (October 1976).

The useful power is the rms value or

$$W_{rms} = 0.707W_{max}$$
 (22)

The input energy is the heat input from which the temperature distribution was obtained (app A).

Physical observations indicate that most of the losses occur in the input of heat to the system.

Equations (3) to (20) may now be cast into a computer program and solved simultaneously without resorting to perturbation or threshold analyses. This simultaneity is important because the actual values of displacements are needed to compute the output power.

4. NUMERICAL RESULTS

The computer program is given in appendix B with a sample printout. Figures 5 to 8 plot the various parameters of interest for typical geometries in which oscillations are self-sustained.

In figure 5, the three liquid displacements, the gas pressure, and the change of gas volume are shown as a function of time. They are in substantial agreement with the descriptive model

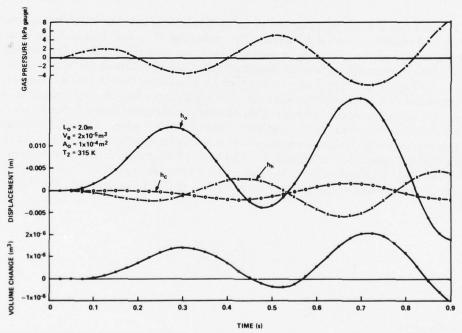


Figure 5. Time history of Fluidyne response: L_0 = liquid inductance output, V_e = volume equilibrium, A_0 = output area, T_2 = temperature of hot side of gas-liquid interface, and h = height of liquid.

proposed as a result of physical observations. Constant temperature apparently causes the oscillations to keep growing. For limit cycling, a more comprehensive, flow dependent temperature distribution has to be developed that in effect provides a time delay in the temperature change. The heat conduction time constant for air is $\tau = \varrho c_p r^2/k \approx 7$ s, where $c_p =$ specific heat at constant pressure (W·s/kgK), r = radius of the pipe (m), and k = thermal conductivity (W/mK). Hence, it is a poor assumption that the temperature reaches the hot or cold value instantaneously, especially since the period of oscillations is an order of magnitude less than 7.0 s. Similarly, the convective time constant due to the movement of the gas-liquid interface must be of the same order, that is, 1 s. Due to the complexity of the fluid mechanic-heat transfer governing equations for such a phenomenon, this development must be left to a more comprehensive and detailed analysis at a later time.

In figure 6, one sees the variation of amplitude and frequency of oscillation for different parametric changes. If one observes the lower three sets of curves, the effect of increasing the output length can be seen. As one would expect, the period increases with output length due to increased inductance.

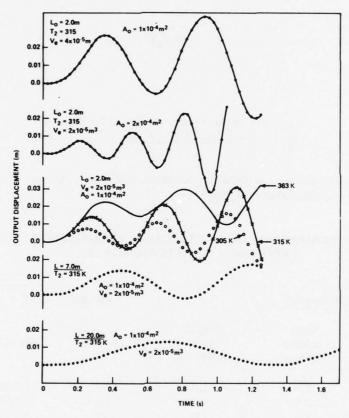


Figure 6. Output displacements for various parameters: L_0 = liquid inertance output, T_2 = temperature of hot side of gas-liquid interface, V_e = volume equilibrium, and A_o = output area.

The middle set of curves demonstrates the effect of increasing the hot side temperature. The frequency decreases slightly with increasing temperature. This decrease is a result of the higher mean pressure in the gas volume that results in a larger volume and, hence, a slightly increased gas capacitance. Interestingly, the amplitude appears to have a maximum at a midrange temperature.

The curve marked "305" shows the effect of increasing the output area. This increase has the effect of decreasing the inductance, again increasing the frequency.

The top curve of figure 6 shows the effect of increasing the inital gas volume and the mass. The gas volume, as already suspected, acts as a capacitance; hence, increasing the capacitance decreases the frequency, as demonstrated in the figure.

Intuitively, one would expect that the predominant frequency determining terms are the output inductance and the gas capacitance. This would lead to the assumption that the system frequency, f, should behave approximately as an RLC circuit, so that

$$f = \frac{1}{2\pi \left(L_{\mathcal{O}}C_{\mathcal{G}}\right)^{\frac{1}{2}}} , \qquad (23)$$

where

$$C_G^{}=$$
 gas capacitance = $V_e/p_\infty^{}$ (m^4s^2/kg) , $p_\infty^{}=$ equilibrium absolute gas pressure (Pa) .

Table I shows the comparison of the frequencies from equation (23) with the actual numerical results obtained from the curves in figure 6.

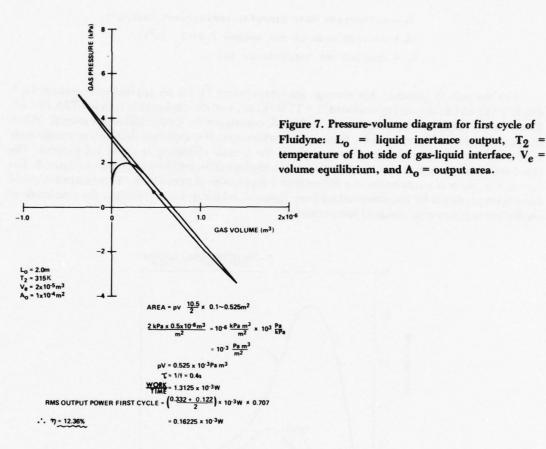
TABLE I. COMPARISON OF NUMERICALLY COMPUTED FLUIDYNE FREQUENCY WITH EQUATION (23)

	Parameter		Frequency (Hz)		
Output length (m)	Gas volume (×10 ⁻⁵ m³)	Output area (×10 ⁻⁴ m²)	Numerical	Eq (23)	Case No.
2.0	2.0	1.0	2.50	2.5299	1
7.0	2.0	1.0	1.54	1.3520	2
20.0	2.0	1.0	0.77	0.7997	3
2.0	4.0	1.0	1.74	1.7883	4
2.0	2.0	2.0	3.33	3.5766	5

Note: Liquid density $\varrho_L = 1000 \text{ kg/m}^3$; ambient absolute pressure $p_{\infty} = 101 \text{ kPa}$.

The agreement is good, so equation (23) may be used as a design guide for determining the operating frequency of a Fluidyne.

The question of cycle efficiency can be answered only by determining the nature of the thermodynamic cycle, the energy input, and the energy output. In figure 7, the relation between gas pressure and volume is shown for startup and the first cycle for case 1 of table I. It is immediately apparent that this is not a Stirling cycle as originally thought by the inventor.' Instead, it



resembles more a Carnot cycle, albeit it is not that, either. The observed efficiency of the Fluidyne's first cycle is 12.36 percent when defined as the output power divided by the power of one cycle ($\int p \, dV$). The heat input to the gas gives the overall efficiency. The heat input may be approximated by the convective heat transfer equation and the mean gas temperature \overline{T}_h as computed in appendix A:

¹C. West, The Fluidyne Heat Engine, United Kingdom Atomic Energy Authority, Harwell, UK, AERE-R-6775 (1971).

$$Q_{in} = HA \left(T_h - \bar{T}_h\right) , \qquad (24)$$

where

H = convective heat transfer coefficient (W/K/m²), A = outside area of hot volume = $2\pi r k_G$ (m²), T_h = applied hot temperature (K) .

For the case in question, the average gas temperature \overline{T}_h for an applied temperature $T_h = 315$ K is 313.454 K. An average value of H = 17 W/K/m², and the gas hot side area is 3.77×10^{-3} m². These values result in a heat input of 0.21 W and, consequently, a poor efficiency overall. When the pump is allowed to operate for three cycles, whereupon it is expected that almost steady-state conditions would normally have been reached, the overall efficiency is up to 1.2 percent. The third-cycle efficiency is shown as a function of the applied differential temperature in figure 8. For $L_{\omega} = 2.0$ m, there is a maximum at a differential temperature of about 24 K. This maximum could have been predicted by the observations from figure 6, where it was shown that the amplitude of oscillation was lower at elevated temperatures.

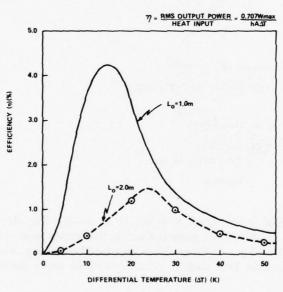


Figure 8. Fluidyne efficiency as function of input temperature and output length: h = height of liquid column and $L_0 = liquid$ inertance output.

Also, it can be observed in figure 6 that the amplitude of oscillation was inversely proportional to the output length; therefore, it seems intuitive that the efficiency increases if the output length decreases. When the gas volume increases, the amplitude also increases, but the frequency decreases. The efficiency may increase; however, since the frequency is lower, the output flow is lower, so the net effect is that the efficiency is not materially affected. Computed results (not shown here) bear out this thesis. Figure 8 shows, however, that efficiency is markedly improved by decreasing the output length. Also shown is that the optimum differential temperature is lowered by a factor of two as the maximum efficiency is more than doubled.

5. DISCUSSION AND CONCLUSIONS

The disturbing point in this analysis is the exponential growth of oscillations. Appendix A discusses this growth and the temperature distribution. More fundamentally, however, the reason for this growth is probably the assumption that the gas changes temperature instantaneously from hot to cold or vice versa. It takes an infinite amount of power or energy to raise a finite amount of mass a finite temperature in zero time. In a numerical model, this infinity is tempered, but still remains large. The conclusion to be drawn, therefore, is that the gas must be much more carefully modeled.

The values of efficiency presented here cannot be used in anything more than a comparative sense since they are based on output power values that are steadily increasing. Observations, however, have indicated that the actual process reaches a steady state after about three cycles. Since the frequencies computed are close to those observed and the amplitudes are reasonable, the three-cycle numerical results have been used.

Two efficiencies are involved, thermodynamic and mechanical. Thermodynamic efficiency is relatively low, and mechanical efficiency, although an order of magnitude greater than thermodynamic, is still only about 10 percent. Thus, the low efficiencies reported are substantiated by this numerical analysis. There is hope, however, of improving the overall efficiency by a factor of two or even three over the extant devices by a thorough parametric study. This study has examined more than 2500 permutations of geometry, and they cannot be presented here. The important observations, however, have been presented, and hope for better efficiency is offered.

Several conclusions may be drawn from this study. The Fluidyne pump operates not, as first thought, on a Stirling cycle, but on a cycle more closely resembling a Carnot cycle. It is essentially a low-efficiency device, although overall thermal efficiencies of 10 percent may conceivably be reached. Poor thermal coupling with the gas accounts for a large portion of this inefficiency. In the real device, it would be advantageous to have a nonconducting liquid and a highly conducting gas (a contradiction of thermodynamic terms). This advantage could possibly be realized by providing an insulating interface substance such as foam. The gas volume should be insulated so that all the heat input stays in the gas.

Relating to the analysis itself, the important points and parameters have been identified. The displacer of the original Fluidyne has been shown to be superfluous to the device. Although amplitude oscillations were never constant, limit cycling might occur when the temperature distribution is corrected or modeled more appropriately. A simple model for the frequency of the

Fluidyne has been proposed that relates the gas volume to the geometry of the output tube in terms of an electrical RLC second-order system analogy. Good agreement with numerical results has been demonstrated.

The overall objectives of this study were to determine if the efficiency of existing Fluidynes could be increased and to comprehensively describe the physics of the Fluidyne operation. These objectives have been met. The efficiency may be increased by appropriate geometric choices, and for the first time a clear description has been proposed for the operation.

Research and development of the Fluidyne heat engine should continue. The Fluidyne can provide fluid power with tiny temperature differentials, with no moving parts. For power production in inaccessible locations, this device is ideal. One of the most inaccessible places is the interior core of the pressure vessel of a nuclear reactor. The nuclear community requires redundant and completely independent safety, monitoring, and control systems. Presently, all backup controls depend on a single type of energy source—electricity. Should this ever fail, then no systems other than the purely mechanical ones could operate. The mechanical systems normally can provide only shutdown capability. A fluidic system, self-contained within the reactor core, with no pressure vessel penetrations, powered by the rectified output of a Fluidyne can provide all the active monitoring and control of the standard systems and also cause shutdown in the event of a scram. This approach can avoid the costly and inconvenient automatic shutdowns presently envisioned. A reactor may continue to operate with the fluidic system installed.

Recent panel discussions sponsored by the American Society of Mechanical Engineers (ASME)* on fluidics and nuclear engineering specifically mentioned fluidic shutdown schemes being developed in the United States. At the same session, fluidic two-diode pumping systems were described that have been developed for the United Kingdom Atomic Energy Authority. These fluidic diode systems have successfully operated with Fluidyne pumps at Harwell. C. F. King of the University College, Cardiff, UK, reported that newly developed vortex diodes that have turndown ratios in excess of 30:1 operate exceptionally well when attached to the Fluidynes. For a session on energetics, fluidic temperature sensors that can withstand extremely hot, harsh environments were reported by Drzewiecki et al. Such sensors are ideal candidates for use with a Fluidyne system for the monitoring of nuclear, fossil fuel, solar, and geothermal power generation plants. Of particular interest are military nuclear power facilities. These are normally mobile and do not have a ready source of auxiliary power to operate secondary systems. Even when they do, the proximity of such auxiliary systems makes them vulnerable to failure due to either the military environment or the radiation possibility from the primary plant itself. The Fluidyne offers a unique solution to this problem. It merely becomes an integral part of the whole.

^{*}T. M. Drzewiecki, Fluidic Applications in Nuclear Engineering, ASME Winter Annual Meeting, Atlanta, GA (November 1977).

⁷T. M. Drzewiecki, R. M. Phillippi, and C. Paras, Fluidics--A New Potential for Energy Conservation by Continuous High Temperature Monitoring and Control, Heat Transfer in Energy Conservation, R. J. Goldstein et al., ed., ASME Book H00106, New York (November 1977), 127-134.

Other military applications for remotely powered fluidic systems are battlefield security and sensor systems (REMBASS). Fluidic sensors have been demonstrated to be uniquely suited to the battlefield. The source of power has been of concern, however. A Fluidyne that can operate on a 3- or 4-deg temperature difference could operate during the day on light energy and during the night on the stored heat in the ground with the colder night air being the thermal sink. This discussion is limited only by imagination. It is the author's opinion that many military systems problems can be effectively and efficiently solved by fluidic, flueric, or fluid systems powered by Fluidyne pumps.

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NOMENCLATURE

- A area (m^2)
- c discharge coefficient (-)
- c = c = c specific heat at constant pressure (W·s/kgK)
- C capacitance (m4s2/kg)
- f frequency (Hz)
- g acceleration of gravity (m/s2)
- h height of liquid column (m)
- H convective heat transfer coefficient (W/K/m2)
- k thermal conductivity (W/mK)
- K constant, =1 when Q < 0; =0 when Q > 0
- l column length (m)
- L liquid inertance (inductance) (kg/m4)
- m mass (kg)
- p absolute pressure (Pa)
- P pressure (Pa)
- Q volumetric flow rate (m³/s)
- r radius of pipe (m)
- R liquid resistance $[kg/(m^4 \cdot s)]$
- R gas constant $[m^3/(s^2K)]$
- t time (s)
- T absolute temperature (K)
- T mean gas temperature
- u liquid velocity (m/s)
- V volume (m³)
- W power (W)
- △ change
- μ fluid absolute viscosity [kg/(m·s)]
- n efficiency
- ρ liquid density (kg/m^3)
- τ time constant (s)
- ω radian oscillating frequency (rad/s)

Subscripts

- c cold
- D displacer
- e equilibrium
- G gas
- h hot
- i integer
- in input
- j integer
- L liquid
- max maximum
 - o output
- rms root mean square average
 - 1 cold side of gas-liquid interface
 - 2 hot side of gas-liquid interface
 - 3 cold side of tube lower end
 - 4 hot side of tube lower end, entrance to output tube node
 - 5 output tube exit
 - ∞ ambient, atmospheric

APPENDIX A.--TEMPERATURE DISTRIBUTION IN AN ENCLOSED CYLINDRICAL GAS VOLUME

The gas volume that provides the means for the motive power in the Fluidyne is essentially cylindrical. It is bounded on either end by liquid interfaces that move so that the volume of gas expands and contracts. Heat is applied to one half and rejected from the other half by maintaining essentially constant hot and cold wall temperatures, T_h and T_c , respectively. Consider, therefore, two touching cylinders with constant wall and end temperatures. The end temperatures can be justified to be T_h or T_c if the conductivity of the liquid is much greater than that of the gas. Figure A-1 shows the assumed geometry.

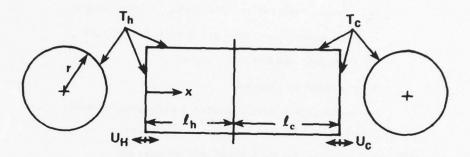


Figure A-1. Schematic of gas volume.

Assume one-dimensional heat flow in the x-direction, and further assume that heat enters the volume by natural convection from the wall temperatures. In the first instance, neglect any motion of the interfaces, and compute the static temperature distribution. For an infinitesimal slice of the volume of length dx, the heat balance is shown in figure A-2.

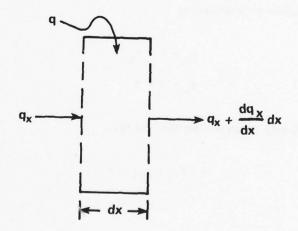


Figure A-2. Heat balance in differential element.

APPENDIX A

The heat balance is

$$q_{x} - \left(q_{x} + \frac{dq_{x}}{dx} dx\right) + q = 0 , \qquad (A-1)$$

where

 q_{x} = heat conducted in x-direction = $-k\pi r^{2} dT/dx$,

q = heat convected into gas = $H2\pi r dx (T_h - T)$,

k = thermal conductivity (W/mK),

r = radius of pipe (m) ,

H = convective heat transfer coefficient (W/K/m²).

The governing differential equations for the hot and cold columns are

$$\frac{d^2T}{dx^2} + \frac{2H}{kr} (T_h - T) = 0 , \qquad (A-2)$$

$$\frac{d^2T}{dx^2} + \frac{2H}{kr} (T_C - T) = 0. (A-3)$$

A new temperature parameter is defined as

$$\theta_{h} = T - T_{h} \quad , \tag{A-4}$$

$$\theta_{C} = T - T_{C} (A-5)$$

In addition, define a length parameter for the cold side,

$$y = (\ell_c + \ell_h) - x , \qquad (A-6)$$

and let

$$\lambda^2 = 2H/kr . (A-7)$$

The governing equations thus become

$$\frac{d^2\theta_h}{dx^2} - \lambda^2\theta_h = 0 , \qquad (A-8)$$

$$\frac{\mathrm{d}^2\theta_c}{\mathrm{d}y^2} - \lambda^2\theta_c = 0 \quad . \tag{A-9}$$

The solutions of equations (A-4) to (A-9) are

$$\theta_h(x) = B_1 \sinh \lambda x + B_2 \cosh \lambda x$$
, (A-10)

$$\theta_{C}(y) = B_{3} \sinh \lambda y + B_{4} \cosh \lambda y$$
 (A-11)

The boundary conditions are

$$T(x=0) = T_{h},$$

$$T(x=\ell_{c}+\ell_{h}) = T_{c},$$

$$T_{h}(x=\ell_{h}) = T_{c}(x=\ell_{h}),$$

$$\frac{dT_{h}(x=\ell_{h})}{dx} = \frac{dT_{c}(x=\ell_{h})}{dx}.$$

In terms of the new parameters,

$$\theta_{h}(0) = 0$$
 , (A-12)

$$\theta_{h}(\ell_{h}) = \theta_{c}(\ell_{c}) + T_{c} - T_{h} , \qquad (A-13)$$

$$\frac{d\theta_{h}}{dx} \left(\ell_{h} \right) = -\frac{d\theta_{c}}{dy} \left(\ell_{c} \right) , \qquad (A-14)$$

$$\theta_{C}(0) = 0$$
 . (A-15)

APPENDIX A

Equations (A-12) and (A-15) make $B_2 = B_4 = 0$. If now ℓ_C is assumed equal to ℓ_h (on the average), then by symmetry equations (A-13) and (A-14) may be replaced if

$$T\left(x=\ell_{h}\right) = \frac{T_{h} + T_{c}}{2} .$$

Thus,

$$\theta_h(\ell_h) = \frac{T_c - T_h}{2}$$

and

$$\theta_{c}(\ell_{c}) = \frac{T_{h} - T_{c}}{2} ,$$

so that

$$\theta_{h}(x) = \frac{\left(T_{c} - T_{h}\right) \sinh \lambda x}{2 \sinh \lambda \ell_{h}} , \qquad (A-16)$$

$$\theta_{C}(y) = \frac{\left(T_{h} - T_{C}\right) \sinh \lambda y}{2 \sinh \lambda \ell_{C}} . \tag{A-17}$$

For the analysis, average values may be used so that

$$\bar{T}_h = T_h + \frac{1}{\ell_h} \int_0^{\ell_h} \theta_h(x) dx$$

and

$$\bar{T}_C = T_C + \frac{1}{\ell_C} \int_0^{\ell_C} \theta_C(y) dy .$$

When equations (A-16) and (A-17) are substituted and the indicated integrations are performed, the results are

$$\bar{T}_{h} = T_{h} + \frac{T_{c} - T_{h}}{2\lambda \ell_{h} \sinh \lambda \ell_{h}} \left(\cosh \lambda \ell_{h} - 1\right)$$
 (A-18)

and

$$\bar{T}_{C} = T_{C} + \frac{T_{h} - T_{C}}{2\lambda \ell_{C} \sinh \lambda \ell_{C}} \left(\cosh \lambda \ell_{C} - 1\right) . \tag{A-19}$$

These average temperatures are used in the numerical model.

A more sophisticated model assumes that the hot side liquid is asymptotically cold. In this model, there are three temperature zones. The formulation is the same for equations (A-18) and (A-19), except that the simplifying assumption of symmetry is not used. The solutions only are given here:

$$\begin{split} T_L(x) &= B_1 e^{\lambda_L x} + T_C \quad , \\ T_h(x) &= B_1 \frac{\lambda_L}{\lambda_G} \quad \sinh \lambda_G x + \left(B_1 + T_C - T_h\right) \cosh \lambda_G x + T_h \quad , \\ T_C(x) &= B_5 \sinh \left[\lambda_G (\ell_C + \ell_h - x)\right] + T_C \quad , \\ B_1 &= \frac{1}{B} \left(T_h - T_C\right) \left[\cosh \lambda_G \ell_h - 1 + \tanh \left(\lambda_G \ell_C\right) \sinh \lambda_G \ell_h\right] \quad , \\ B &= \frac{\lambda_L}{\lambda_G} \sinh \lambda_G \ell_h + \cosh \lambda_G \ell_h + \frac{1}{\lambda_G} \tanh \lambda_G \ell_C \left(\lambda_L \cosh \lambda_G \ell_h + \lambda_G \sinh \lambda_G \ell_h\right) \quad , \\ B_5 &= \frac{B_1 \left(\frac{\lambda_L}{\lambda_G} \sinh \lambda_G \ell_h + \cosh \lambda_G \ell_h\right) + \left(T_C - T_h\right) \left(\cosh \lambda_G \ell_h - 1\right)}{\sinh \lambda_G \ell_C} \quad , \end{split}$$

and the average hot and cold temperatures are

$$\begin{split} \overline{T}_{h} &= T_{h} + \frac{B_{1}\lambda_{L}}{\lambda_{G}^{2}\ell_{h}} \left(\cosh \lambda_{G}\ell_{h} - 1\right) + \frac{B_{1} + T_{C} - T_{h}}{\lambda_{G}\ell_{h}} \sinh \lambda_{G}\ell_{h} \quad , \\ \\ \overline{T}_{C} &= T_{C} + \frac{B_{5}}{\lambda_{G}\ell_{C}} \left(\cosh \lambda_{G}\ell_{C} - 1\right) \quad , \end{split}$$

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where

 T_L = temperature of liquid , G = gas , ℓ_C = length of cold column , ℓ_h = length of hot column , ℓ_h = constant of integration .

Even more sophistication may be introduced by dropping the one-dimensional restriction. Dropping it leads to two governing equations,

$$\frac{\partial^2 \theta_{\underline{i}}}{\partial \xi^2} + \left(\frac{\ell_h + \ell_c}{R}\right)^2 \frac{\partial^2 \theta_{\underline{i}}}{\partial \psi^2} + \left(\frac{\ell_h + \ell_c}{R}\right)^2 \frac{1}{\psi} \frac{\partial^2 \theta_{\underline{i}}}{\partial \psi} = 0 ,$$

where

$$\theta_{i} = \frac{T - T_{C}}{T_{h} - T_{C}} ,$$

$$\xi = x / (\ell_{h} + \ell_{c}) ,$$

$$R = \text{liquid resistance},$$

$$\psi = r/R,$$

$$i = L, G,$$

with these boundary conditions on the liquid:

$$\theta_{L}(\xi, 1) = 0 ,$$

$$\theta_{L}(-\infty, \psi) = 0 ,$$

$$\frac{\partial \theta_{L}}{\partial \psi} (\xi, 0) = 0 ,$$

$$\theta_{L}(0, \psi) = \theta_{G}(0, \psi) ;$$

and with these boundary conditions on the gas:

$$\theta_{G}(\xi, 1) = \begin{cases} 1, & 0 \leq \xi < \ell_{h} / (\ell_{h} + \ell_{c}) \\ 0, & \xi \geq \ell_{h} / (\ell_{h} + \ell_{c}) \end{cases},$$

$$\frac{\partial \theta_{G}}{\partial \Psi}(\xi, 0) = 0 ,$$

$$\frac{\partial \theta_{G}}{\partial \xi}(0, \Psi) = \frac{\partial \theta_{L}}{\partial \xi}(0, \Psi) ,$$

$$\theta_{G}(1, \Psi) = 0 .$$

These equations may be solved by separation of variables and lead to two eigenvalue problems resulting in a double infinite series in hyperbolic functions and Bessel functions. This tedious solution is left for the reader if he should desire to pursue it.

The original differential equations (A-2) and (A-3) are altered when gaseous motion is considered. Equation (A-3) becomes

$$\frac{d^2T}{dx^2} + \frac{2H}{Rk} \left(T_h - T \right) - \frac{\Psi_C u}{k} \frac{dT}{dx} = 0 , \qquad (A-20)$$

where u = liquid viscosity. If one assumes that the initial static temperature distribution is developed prior to any volume change (gas velocity onset), equation (A-20) may be used to examine the relative importance of the convective and conductive terms. Assume that a temperature distribution exists. Assign order of magnitude values to the derivatives:

$$\begin{split} \frac{dT}{dx} &\sim \frac{^T\!h}{^L} - ^T\!c \\ \frac{d^2T}{dx^2} &\sim \frac{^T\!h}{^L} - ^T\!c \\ \frac{d^2T}{dx^2} &\sim \frac{^T\!h}{^L} - ^T\!c \\ \end{split} \ . \label{eq:dtdx}$$

For the conductive term $d^{z}T/dx^{z}$ to be negligible compared with the convective term $(\Psi c_{p}u/k)\,dT/dx,$

$$\frac{d^2T}{dx^2} << \frac{\psi c_p u}{k} \frac{dT}{dx}$$

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or

$$\frac{\Psi c u l_h}{k} >> 1 \qquad (A-21)$$

For air,

$$\psi = 1.2 \text{ kg/m}^3 ,$$

$$c_p = 1.05 \times 10^3 \text{ W·s/kgK, specific heat at constant pressure,}$$

$$\ell_h = 0.1 \text{ m ,}$$

$$k = 0.026 \text{ W/mK, thermal conductivity .}$$

For the velocity, assume sinusoidal displacement of ± 0.02 m at 3 Hz, so that the maximum velocity is $2\pi f(0.02) = 0.38$ m/s. Therefore,

$$\frac{\Psi c_p u \ell_h}{k} = 1841 >> 1 .$$

This indicates that during oscillation there is no significant effect of conduction; consequently, if the gas volume moves back and forth as a slug, the temperature is governed only by

$$\frac{d\theta}{dx} + \frac{\Psi_{C} uR}{2H} \theta = 0 ,$$

which implies that

$$\theta = B_6 \exp \left(-\frac{\Psi c uR}{2H} x\right)$$
,

and if the boundary condition is that $\theta = 0$, then $B_6 = 0$, and the temperature remains at its initial constant value.

In reality, the volume expands and contracts, and there is conduction with a lag time, so that the initial temperature distribution eventually decays to a more uniform overall temperature due to the mixing effect of the convective terms. In addition, more complex boundary conditions exist. A thermal lag through the wall or the device supplies heat, and there is conduction axially in the walls. A considerable effort would have to be expended to thoroughly analyze the complete problem, given the velocity distribution. If we add now the complexity of coupling the transient

APPENDIX A

three-dimensional energy equation and the Navier-Stokes equations for the gas to the time-dependent oscillating liquid equations, the problem becomes massive.

A tentative time to solve such a problem is 1 to 2 man years. Such a solution would be highly desirable if any great effort or expenditure were ever contemplated for the Fluidyne.

APPENDIX B.--COMPUTER PROGRAM AND NUMERICAL RESULTS

This appendix presents the Fluidyne pump computer program written in FORTRAN IV for a 360/70 IBM computer. Five first-order differential equations are integrated by DVERK, an IBM supplied Runge-Kutta scheme that uses a fifth- and a sixth-order algorithm to reduce the global error to that specified by the user. It chooses the time increments within the desired steps of printout, which are specified by XEND. The program has five nested parameter DO loops that compute the response of the Fluidyne while the following parameters are swept:

Output length
Equilibrium liquid height
Equilibrium gas volume
Output area
Maximum hot temperature

All units are specified in SI units, except for the gas constant, which is in English customary units, but is converted in the program. All outputs are in SI units.

A sample printout is given for case 1 of table I in the main body of the report.

APPENDIX B

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                                                                                                                                                                                                                                                                                                                                                                                                                                                   NAVAL PISTGRACLATE SCHOOL
          FILE: FLLEYN FURTRAN PI
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          FLU0003250
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: FLUDYN FORTRAN P1 NAVAL POSTGRACLATE SCHOOL

TC=TI+(12-T1)*F
TH=T2+(T1-T2)*F
P1=GMASS*R*TC*TH/(V1*TH+V2*TC) FLU007720
P2=P1
IF(Y(1).GT.0.0) GC TC 6 FLU007730
B1=1.0
GO TO 7
B1=0.0
CONTINUE
P4=H24*(-YPRIME(1)-YPRIME(2))+P2+R24*(-Y(1)-Y(2))+R+CLIQ/2.*(Y(1)/FLUC0780)
P3=P4-RHCLIG/.72*Y(2)*ABS(Y(2))/(A24-A45)**2
CALL DVERK(N,FCN,X,Y,XEND.TOL,IND,C,NW,W,IER)
IF(INC.LT.0.GR.IER.GT.0) GO TO 20 FLU00830
P3=Y4-Y(4)-HE
Y5=Y(5)-HE
Y5=Y(5)-HE
P7=P6-P5*.OG1
V4=V4-VE
WRITE(6,30) Y(1).POWER,Y3,Y4.Y5,X,P7,T2.V4
FLU00890
P7=P1-DELT
FLU00990
T2=TY-DELT
FLU00990
T2=TY-DELT
FLU00990
T2=TY-DELT
FLU00990
T1=TWAX
CONTINUE
WRITE(6,87) TC,TH.R.D.F
WRITE(6,87) TC,TH.R.D.F
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FRITE(6,87) TC,TH.R.D.F
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  FILE: FLUDYN FORTRAN P1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                NAVAL POSTGRACLATE SCHOOL
  9
  10
                                                              WRITE(6,88) TC,TH,K,D,F

WRITE(6,88) TC,TH,K,D,F

WRITE(6,88) FORMAT(1) TC=1,F8.3, TH=1,F8.3, R=1,F10.3, D=1,E10.4, F=1,E10.4FLU005960

FLU01000

FLU010100

CONTINUE

SIDP

TIME GAS PRESSURE HOT TEMP GAS VOLUME')

FORMAT(1213)

FORMAT(1213)

CONTINUE

WRITE(6,40) IER,INC

FORMAT(213)

STOP

FU01160

FLU01120

FLU01120

FLU01120

FLU01120

FLU01120

FLU01120

FLU01120

FLU01230

FLU01230

FLU01240

FLU01230

FLU01240

FLU01240

FLU01250

FLU01250

FLU01250

FLU01260

FLU01260

FLU01270

FLU01280

FLU01310

FLU01310

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